

Exercise 36Calculate y' .

$$y = \sqrt{t \ln(t^4)}$$

SolutionCalculate y' by using the chain and product rules.

$$\begin{aligned}y' &= \frac{d}{dt} \sqrt{t \ln(t^4)} \\&= \frac{d}{dt} \sqrt{4t \ln t} \\&= \frac{d}{dt} 2\sqrt{t \ln t} \\&= 2 \frac{d}{dt} \sqrt{t \ln t} \\&= 2 \left(\frac{1}{2} \right) (t \ln t)^{-1/2} \cdot \frac{d}{dt} (t \ln t) \\&= (t \ln t)^{-1/2} \cdot \left\{ \left[\frac{d}{dt} (t) \right] \ln t + t \left[\frac{d}{dt} (\ln t) \right] \right\} \\&= \frac{1}{\sqrt{t \ln t}} \cdot \left[(1) \ln t + t \left(\frac{1}{t} \right) \right] \\&= \frac{1}{\sqrt{t \ln t}} \cdot (\ln t + 1) \\&= \frac{\ln t + 1}{\sqrt{t \ln t}}\end{aligned}$$